**Formula Sheet**

**Chapter 2**

**Definition 2.1 Experiment**

An experiment is the process by which an observation is made.

**Definition 2.2 Simple Event**

A simple event is an event that cannot be decomposed. Each simple event corresponds to one and only one sample point. The letter E with a subscript will be used to denote a simple event or the corresponding sample point.

**Definition 2.3 Sample Space**

The sample space associated with an experiment is the set consisting of all possible sample points. A sample space will be denoted by S.

**Definition 2.4 Discrete Sample Space**

A discrete sample space is one that contains either a finite or a countable number of distinct sample points.

**Definition 2.5 Event**

An event in a discrete sample space S is a collection of sample points—that is, any subset of S.

**Definition 2.6 Probability**

Suppose S is a sample space associated with an experiment. To every event A in S (A is a subset of S), we assign a number, P(A), called the probability of A, so that the following axioms hold:

Axiom 1:

Axiom 2:

Axiom 3: If A1, A2, A3, . . . form a sequence of pairwise mutually exclusive events in S (that is, if ) then

**Sample-point Method**

The sample-point method is outlined in Section 2.4. The following steps are used to find the probability of an event:

1. Define the experiment and clearly determine how to describe one simple event.

2. List the simple events associated with the experiment and test each to make certain that it cannot be decomposed. This defines the sample space S.

3. Assign reasonable probabilities to the sample points in S, making certain that P(Ei) ≥ 0 and P(Ei) = 1.

4. Define the event of interest, A, as a specific collection of sample points. (A sample point is in A if A occurs when the sample point occurs. Test all sample points in S to identify those in A.)

5. Find P(A) by summing the probabilities of the sample points in A.

**Theorem 2.1 mn = m x n**

With m elements a1, a2,..., am and n elements b1, b2,..., bn, it is possible to form mn = m × n pairs containing one element from each group.

**Definition 2.7 Permutation**

An ordered arrangement ofr distinct objects is called a permutation. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol Pn r .

**Definition 2.8 Combinations**

The number of combinations of *n* objects taken *r* at a time is the number of subsets, each of size *r*, that can be formed from the *n* objects. This number will be denoted by or ()

**Definition 2.9 Conditional Probability of an Event**

The conditional probability of an event A, given that an event B has occurred, is equal to

provided P(B) > 0. [The symbol P(A|B) is read “probability of A given B.”]

**Definition 2.10 Independent Cases**

Two events A and B are said to be independent if any one of the following holds:

Otherwise, the events are said to be dependent.

**Theorem 2.5 The Multiplicative Law of Probability**

The Multiplicative Law of Probability The probability of the intersection of two events A and B is

If A and B are independent, then

**Theorem 2.6 The Additive Law of Probability**

The Additive Law of Probability The probability of the union of two events A and B is

If A and B are mutually exclusive events, P(A ∩ B) = 0 and

**Theorem 2.7 Mutual Exclusive Events**

If A is an event, then

**Definition 2.11**

For some positive integer k, let the sets B1, B2,..., Bk be such that

1. S = B1 ∪ B2 ∪···∪ Bk .

2. Bi ∩ Bj = ∅, for i = j.

Then the collection of sets {B1, B2,..., Bk } is said to be a partition of S.

**Theorem 2.8**

Assume that is a partition of S(see Definition 2.11) such that Then for any event A

**Theorem 2.9 Bayes’ Rule**

Bayes’ Rule Assume that {B1, B2,..., Bk } is a partition of S (see Definition 2.11) such that P(Bi) > 0, for i = 1, 2,..., k. Then

**Definition 2.12 Random Variable**

A random variable is a real-valued function for which the domain is a sample space

**Definition 2.13 Random Sample**

Let N and n represent the numbers of elements in the population and sample, respectively. If the sampling is conducted in such a way that each of the N n samples has an equal probability of being selected, the sampling is said to be random, and the result is said to be a random sample.

**Chapter 3**

**Definition 3.1 Discrete Values**

A random variable Y is said to be discrete if it can assume only a finite or countably infinite1 number of distinct values.

**Definition 3.2 Sum of the Probabilities of all Sample Points**

The probability that Y takes on the value y, P(Y = y), is defined as the sum of the probabilities of all sample points in S that are assigned the value y. We will sometimes denote P(Y = y) by p(y).

**Definition 3.3 Probability Distribution**

The probability distribution for a discrete variable Y can be represented by a formula, a table, or a graph that provides p(y) = P(Y = y) for all y.

**Definition 3.4 Expected Value**

Let Y be a discrete random variable with the probability function p(y). Then the expected value of Y , E(Y ), is defined to be2

**Definition 3.5 Random Variable w/Mean**

If Y is a random variable with mean E(Y ) = µ, the variance of a random variable Y is defined to be the expected value of (Y − µ)2. That is,

The standard deviation of Y is the positive square root of V(Y ).

**Definition 3.6 Binomial Experiment**

A binomial experiment possesses the following properties:

1. The experiment consists of a fixed number, n, of identical trials.

2. Each trial results in one of two outcomes: success, S, or failure, F.

3. The probability of success on a single trial is equal to some value p and remains the same from trial to trial. The probability of a failure is equal to q = (1 − p).

4. The trials are independent.

5. The random variable of interest is Y , the number of successes observed during the n trials.

**Definition 3.7 Binomial Distribution**

A random variable Y is said to have a binomial distribution based on n trials with success probability p if and only if

, and

**Definition 3.8 Geometric Probability**

A random variable Y is said to have a geometric probability distribution if and only if

**Definition 3.9 Negative Binomial Probability**

A random variable Y is said to have a negative binomial probability distribution if and only if

**Theorem 3.9 Negative Binomial Probability**

If Y is a random variable with a negative binomial distribution,

and

**Definition 3.10 Hypergeometric Probability**

A random variable Y is said to have a hypergeometric probability distribution if and only if

where y is an integer 0, 1, 2,…, n, subject to the restrictions and

**Theorem 3.10 Hypergeometric Probability**

If Y is a random variable with a hypergeometric distribution,

and )

**Definition 3.11 Poisson Probability**

A random variable Y is said to have a Poisson probability distribution if and only if

**Theorem 3.11 Poisson Probability**

If Y is a random variable possessing a Poisson distribution with parameter , then

**Theorem 3.14 Tchebysheff’s Theorem**

Let Y be a random variable with mean and finite variance Then, for any constant

**Chapter 4**

**Definition 4.1**

Let Y denote any random variable. The *distribution function* of Y, denoted by F(y), is such that

for

**Theorem 4.1 Properties of a Distribution Function**

If F(y) is a distribution function, then

1. is a nondecreasing function of y. [If and are *any* values such that then

**Definition 4.2**

A random variable Y with distribution function F(y) is said to be *continuous* if F(y) is continuous, for

**Definition 4.3**

Let F(y) be the distribution function for a continuous random variable Y. Then f(y), given by

wherever the derivative exists, is called the *probability density function* for the random variable Y.

**Theorem 4.2 Properties of a Density Function**

If f(y) is a density function for a continuous random variable, then

**Definition 4.4**

Let Y denote any random variable. If Y, denoted by , is the smallest value such that P(Y If Y is continuous, is the smallest value such that F( Some prefer to call the 100*p*th *percentile* of Y.

**Theorem 4.3**

If the random variable Y has density function f (y) and a < b, then the probability that Y falls in the interval [a, b] is

**Definition 4.5**

The expected value of a continuous random variable Y is

provided that the integral exists.

**Theorem 4.4**

Let g(Y) be a function of Y; then the expected value of g(Y) is given by

provided that the integral exists.

**Theorem 4.5**

Let c be a constant and let g(Y), be functions of a continuous random variable Y. Then the following results hold:

**Definition 4.6**

If , a random variable Y is said to have a continuous *uniform probability distribution* on the interval ( if and only if the density function of Y is

**Definition 4.7**

The constants that determine the specific form of a density function are called *parameters* of the density function.

**Theorem 4.6**

If and Y is a random variable uniformly distributed on the interval (, then

and

**Chapter 5**

**Definition 5.1**

Let be discrete random variables. The *joint* (or bivariate) *probability function* for is given by

**Theorem 5.1**

If are discrete random variables with joint probability function

1. where the sum is over all values that are assigned nonzero probabilities.

**Definition 5.2**

For any random variables the joint (bivariate) distribution function is

,

**Definition 5.3**

Let and be continuous random variables with joint distribution function If there exists a nonnegative function , such that

for all

is called the *joint probability density function.*

**Theorem 5.2**

If are random variables with joint distribution function then



**Theorem 5.2**

If are jointly continuous random variables with joint density function given by then

**Definition 5.4**

1. Let be jointly discrete random variables with probability function p( Then the *marginal probability functions* of , respectively, are given by
2. Let be jointly continuous random variables with joint density function f( Then the *marginal density functions* of , respectively, are given by

**Definition 5.5**

If are jointly discrete random variables with joint probability function and marginal probability functions and , respectively, then the *conditional discrete probability function* of is

provided that

**Definition 5.6**

If are jointly continuous random variables with joint density function , then the *conditional distribution function of* is

.

**Definition 5.7**

If are jointly continuous random variables with joint density and marginal densities and , respectively. For any such that the conditional density of given is given by

and, for any such that (> 0, the conditional density of given is given by

**Definition 5.8**

Let have distribution function have distribution function and have joint distribution function F ( Then and are said to be *independent* if and only if

for every pair of real numbers (

If and are not independent, they are said to be *dependent.*

**Theorem 5.4**

If and are discrete random variables with joint probability function p and marginal probability functions and , respectively, then and are independent if and only if

for all pairs of real numbers (.

If and are continuous random variables with joint density function f and marginal density functions and , respectively, then and are independent if and only if

for all pairs of real numbers (.

**Theorem 5.5**

Let and have a joint density f that is positive if and only if otherwise. Then are independent random variables if and only if

where ( is a nonnegative function of alone and h( is a nonnegative function of alone.